Case Study for the Mathematical Modelling and Problem Solving Project

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Abstract

The School of Mathematics at the University of Manchester was one of the original partners in the Mathematical Modelling and Problem Solving project led by the University of Leeds. Our objectives were

- to introduce mathematical modelling into the first year Mathematical Workshop course
- to introduce Matlab within the Mathematical Workshop to develop basic programming skills and provide a tool to solve modelling problems
- to identify current modelling activity in our undergraduate programmes
- to create a discussion in the School to share good practice and recognise the importance of modelling
- to develop a culture among staff and students of the importance of modelling in mathematics
- to engage with local schools and colleges to help them understand the importance of modelling

This report describes the how we have implemented these objectives and to what extent they have been successful.

Background and Rationale

Mathematical modelling and problem solving are extremely useful skills that we wish our undergraduate students to develop to help them apply their mathematical knowledge in real life situations. Our undergraduate curriculum includes several courses with 'modelling' in the title but the emphasis is on solving given models rather than developing and analysing these models.

The Mathematical Workshop has been running for several years and has proved an effective way to develop students' team working and problem solving skills using group project work. However these projects have focused on developing mathematical skills rather than modelling real world problems.

We have built strong links with graduate employers through our careers and employability activities. These employers require evidence of teamworking, open ended problem solving and programming skills from graduate applicants. Helping students to develop these skills will become increasingly important in a highly competitive graduate jobs market. Mathematical modelling can be a vehicle to deliver essential skills that students require, such as group work, communication, and problem solving skills. These are all things that employers appreciate and at the current time our undergraduate curriculum is limited in its scope for developing these qualities.

Mathematical modelling goes right to the heart of most applied mathematics today, whether that is in finance, fluid dynamics or statistics. Increasing the amount of modelling we offer to our undergraduates could vastly increase awareness of contemporary mathematical research and develop key skills to allow help students apply mathematics in the workplace or in further study. It could help develop the ability of undergraduates to draw on all areas of their degree programme and to tackle unseen real world problems.

Implementation

In this section we describe the activities undertaken as part of the project.

1. The Mathematical Workshop

This course is taken by nearly 300 single honours mathematics students in the first semester of the first year. It consists of a one hour lecture and a two hour workshop each week. This academic year the course was changed substantially to include an introduction to Matlab and mathematical modelling. The workshop is either held in a computer cluster or in a classroom where the students work in groups of 5 or 6 with a postgraduate student facilitator. The weekly breakdown is as follows:

Week 1: introduction to the course
Week 2-5: introduction to Matlab and numerical methods
Week 6: mid semester break
Week 7: introduction to mathematical modelling
Week 8-9: project 1
Week 10-11: project 2
Week 12: Test

Full details of the course and materials can be found at
http://www.maths.manchester.ac.uk/undergraduate/ugstudies/units/2011-12/level1/MATH10001/
The assessment of the course is

Matlab workshop exercises (10%)
Matlab project (20%)
Two project reports (30% each)
In-class test (10%).

The marking of projects is done by the postgraduate facilitators. Typically students' received their marked work back a week after they submitted it. Although students work on the problems in groups during the second half of the course, the project reports are written and submitted individually. This gives the students a chance to explain their interpretation of the problems and helps them to develop their written communication skills. Marks are awarded for the quality of their explanations as well as the correctness. There is also a group mark based on the average mark for members of the group who have taken part in the workshops. This encourages participation in the workshop sessions.

2. Investigation of modelling in our current programmes, creating a discussion among staff and students about the importance of mathematical modelling and suggestions for future developments

We held discussions with staff and students to characterize the current state of mathematical modelling in the undergraduate programmes offered by the School and to identify ways in which it could be improved. This was done by employing two project developers (one postgraduate student who had also studied as an undergraduate here and a final year undergraduate student) to undertake a series of interviews with academic staff and students and a focus group with undergraduate students. Two other postgraduate students were employed to investigate and develop ideas for modelling projects. We held a teaching seminar for academic staff to disseminate the findings of the project.

When asked about the benefits that modelling could bring, most lecturers could see great

benefits to both academia and to employment; they often see many unprepared PhD students in respect of their programming abilities and modelling skills and the view was expressed by one member of staff that:

> "computing and modelling are the best employment skills that a mathematics degree offers."

It was noted that undertaking a modelling project or taught course would allow students to gain core transferrable skills combined with mathematical skills and would genuinely enable students to use mathematics to solve real world problems.

Many lecturers are keen to see computer programming running in parallel with mathematical modelling and they believe that is should be a compulsory part of the undergraduate syllabus. Several lecturers already include some Matlab and Mathematica exercises in their courses but we need to raise awareness of the wider benefits.

A small minority of lecturers believe that there is already enough modelling in the undergraduate syllabus or that an undergraduate mathematics degree is not a natural place to teach modelling. However this view was strongly outweighed by the majority of staff and students who feel that although there is some modelling in the 3rd year applied courses such as *MATH35001 Viscous Fluid Flow*, and *MATH35021 Elasticity*, the main part of this is working within an existing model and not comparing or testing other models or developing new models. On the whole most people would like to see the introduction of a new module in the second semester 2nd year or sometime in the 3rd year, or a staged progression beginning with *MATH10001 Mathematical Workshop* and continuing over the whole degree programme.

Students felt that some modules in the existing course structure which could be modified to include more modelling for instance *MATH39032 Mathematical Modelling in Finance*. Here students thought that the first half of the course could be compressed and that would leave 4 weeks to complete a modelling project. There are other modules with sufficient modelling content such as *MATH20712 Random Models* and *MATH49111 Scientific Computing.*

At the focus group we discussed a project that is organized in China by the China Society for Industrial and Applied Mathematics. They facilitate a competition called the Contemporary Undergraduate Mathematical Contest [*http://en.mcm.edu.cn*] which involves teams of three students from university with an academic advisor competing to develop a mathematical or computer model for a real world problem.

With the introduction of the new MSc in Applied Mathematics in 2012-13 there will be a new course unit called Transferable Skills for Applied Mathematicians, this will involve both modelling and programming and this could provide a good indication of how to implement a modelling module for the undergraduate courses. Some issues would need to be addressed, such as how to deal with a large class size, the assessment style and where to locate it in the programme.

3. Engagement with local schools and colleges to emphasise the importance of mathematical modelling and problem solving

We offer a broad range of events to engage school and college students in mathematics and its applications. As well as interesting mathematical activities and talks we are increasingly including presentations on the use of mathematics in the real world and the career opportunities open to students who study mathematics at university.

Our two day residential summer school, Making Maths at Manchester, is a chance for year 12 students to work in groups on open ended mathematical problems. They are introduced to new ideas and approaches that are very different to school mathematics.

As well as these established events we have been working with MEI to promote the A-level extended project. This is an excellent way for students to develop modelling and problem solving skills and to become independent, reflective learners. At the moment there are very few extended projects in mathematics and these often focus on an historical topic. This may be because students are unaware of the wide range of applications of mathematics in the real world. To address this we organised a full day workshop on 21st June 2012 to highlight suitable projects and to describe the support available to teachers and students undertaking a project. We had ten teachers and over 40 students attend the event. After an introductory talk on the applications of mathematics in the real world, the students worked in groups on suggested topics for the rest of the day. There was a separate session for teachers where they were given advice on supporting projects and useful resources. The University of Manchester offers support through online resources and access to the library and staff in the School of Mathematics will offer expert advice on individual projects.

Evaluation

The formal evaluation of the project is being done by a team from the School of Education at the University of Manchester led by Prof. Julian Williams. The results and analysis will be detailed separately and so we will only briefly describe the evaluation of our part of the project here.

All students taking the Mathematical Workshop were evaluated using a tool developed by the team. This was done in week 7 of the course when the students were first introduced to mathematical modelling and again at the end of the course.

The postgraduate facilitators helped in marking this evaluation tool and were interviewed about there involvement in the course. We held a focus group in February 2012 with seven of the undergraduate students who had taken the Mathematical Workshop. They were asked to talk about their projects, what they had gained from the course and the teaching and learning methods.

The course lecturer was interviewed and this together with videos of the focus group, lectures and workshops has been edited into a short film about the project.

Prof. Williams attended the extended project workshop and will be including this in the evaluation. Feedback back from the event from both students and teachers was favourable and we intend to run this workshop on an annual basis.

Discussion, Learning and Impact

On the whole most lecturers and students really like the idea of increasing the amount of modelling in our courses. Students need to learn how to look at a problem, how to formulate a model and then how to solve it using simple methods. The skills gained from learning to create, review and modify mathematical models can be applied to many other course units and these skills are as important as the mathematics involved.

At present students do not believe they are taught about how to model in their courses and they would like to see much more emphasis on modelling. They think it would be valuable in terms of employability skills and would be an interesting variation to the normal taught lecture course. Students would also like to see an increased amount of group work, and report writing and they think that completing some coursework based modelling would be a good way to do this.

Staff and students alike strongly feel that a mathematics degree should not be about learning lots of content that is assessed and then forgotten. Instead we should be teaching students what to do with the mathematics they are taught and the skills they need to solve unfamiliar problems. Most students think that being able to apply the mathematics they have learned to a real world problem is an essential skill and they would appreciate the opportunity to develop modelling techniques on their degree. From their experience of using MATLAB in the first year students realise the importance of programming and would like to see more small group teaching/ facilitation in this area.

Introducing programming and modelling skills into the Mathematical Workshop has emphasised the importance of these skills to first year students who have little previous experience of this approach to mathematics. However it is important that there is a continued emphasis on the importance of modelling and problem solving throughout their degree programme with ample opportunities to become aware of applications of mathematics and to practice modelling in a broad range of mathematical course units. Academic staff are often engaged in mathematical modelling as part of their research and therefore do not need to be persuaded of the importance and relevance of these skills. However it can be challenging to develop new ways to engage undergraduate students in the full modelling cycle as this requires knowledge of areas that fall outside a traditional mathematics curriculum.

Over the course of this project we identified several key barriers that would need to be addressed to increase the level of mathematical modelling in our undergraduate courses. From a practical point of view there is a key theme that there is not enough room to fit modelling into a current course unit and that lecturers would not be prepared to lose any existing content from their courses, however they would be keen to see a new module introduced.

Through investigations with staff and students we believe that it is desirable that modelling should be offered in a group work setting. One lecturer said:

> "I only know one way to teach modelling and that is by getting them [students] to do it – they require encouragement and feedback."

Lecturers suggested that modelling works very well in groups as different people have different parts of the puzzle, and often it can act as a boost to weaker students who can draw on the support of other members of their team. A staff member said:

> "It is a bad thing that we don't have group work – no presentations, or report writing."

However group work raises the issue of how to fairly assess an individual student's contribution to the project.

Project based courses tend to involve a high workload for academic staff with the need to think up new projects every year and marking of reports. Involving PhD students as group facilitators would bring benefits to both student and facilitator and could relieve the pressures on a course leader by spreading the workload.

Further Development and Sustainability

We will continue to run the Mathematical Workshop in its current form for the next few years. However it is essential that students have an opportunity to further develop their modelling and problem solving skills at a higher level. To this end we intend to introduce a second year course in mathematical modelling. This will involve group project work where students develop, solve and evaluate models based on real world problems. We are in discussion with a small group of academics to develop this course in the coming academic year so that it can be delivered in 2013-14. A similar course for MSc students will run in 2012-13 and we will be able to learn valuable lessons from this in terms of delivery and assessment.

The A-level extended project workshop will hopefully lead to several of the participating students taking up a project in the next academic year. We have offered to support these students by offering expert advice on project material as well as access to the library and online resources.

The introduction of a 'modelling club' based on those described by some of our Chinese students is something we will investigate in the coming months. This could involve visiting speakers from industry presenting a problem and then groups of students and academics working to solve the problem.

Outputs

The materials developed in this project will be made available online.